Ultrafast electrical spectrum analyzer based on all-optical Fourier transform and temporal magnification

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Abstract: Real-time electrical spectrum analysis is of great significance for applications involving radio astronomy and electronic warfare, e.g. the dynamic spectrum monitoring of outer space signal, and the instantaneous capture of frequency from other electronic systems. However, conventional electrical spectrum analyzer (ESA) has limited operation speed and observation bandwidth due to the electronic bottleneck. Therefore, a variety of photonics-assisted methods have been extensively explored due to the bandwidth advantage of the optical domain. Alternatively, we proposed and experimentally demonstrated an ultrafast ESA based on all-optical Fourier transform and temporal magnification in this paper. The radio-frequency (RF) signal under test is temporally multiplexed to the spectrum of an ultrashort pulse, thus the frequency information is converted to the time axis. Moreover, since the bandwidth of this ultrashort pulse is far beyond that of the state-of-the-art photodetector, a temporal magnification system is applied to stretch the time axis, and capture the RF spectrum with 1-GHz resolution. The observation bandwidth of this ultrafast ESA is over 20 GHz, limited by that of the electro-optic modulator. Since all the signal processing is in the optical domain, the acquisition frame rate can be as high as 50 MHz. This ultrafast ESA scheme can be further improved with better dispersive engineering, and is promising for some ultrafast spectral information acquisition applications.

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References and links
1. Introduction

Electrical spectrum analyzer (ESA) is a fundamental instrument widely applied in a variety of fields including the wireless communication, the radar system, and the radio astronomy [1–3]. Conventional super-heterodyne spectrum analyzer can achieve hyperfine resolution and large observation bandwidth, but the operation speed is mainly restricted by the sweep time of the local oscillator. As an improvement, the fast Fourier transform (FFT) based ESA greatly enhanced the measurement speed while maintaining a finer resolution. However, the observation bandwidth of this ESA system is inherently limited by that of the analog-to-digital converter (ADC) which is essential for the FFT process [4]. Therefore, the conventional ESA based on electronic technology is hard to further extend the observation bandwidth and the acquisition frame rate, which is undesirable in many applications where ultrafast analysis or large bandwidth measurement is required. Fortunately, photonics-assisted spectrum analysis approaches have arisen with the advanced microwave photonics, which leverages the large bandwidth in the optical domain, and multiplexes the radio-frequency (RF) to the optical field by an electro-optic modulator (EOM) [5]. These RF measurement approaches are able to achieve large bandwidth or fast acquisition frame rate, based on different mechanisms, such as optical power monitoring [6–9], optical channelizing [10–13], time-domain optical processing like time stretch [14,15] and all-optical Fourier transform [16,17], and compressed sensing (CS) [18–20]. The optical power monitoring...
maps the RF frequency to the output power, thus the frequency can be identified by measuring the optical power. Although a fine resolution and wide bandwidth can be achieved, this kind of method can only characterize a single frequency component. Alternatively, the optical channelizer approach is usually implemented with the RF source multiplexed to an optical carrier that its spectrum is up converted to the optical band, thus, can be resolved by an optical spectrum analyzer, namely the optical channelizer, which maybe a Fabry-Perot etalon [10], a array waveguide gratings (AWG) [11], or a diffraction grating [12]. Owing to the limited resolving power of the optical channelizer, the resolution of this approach is always exceed 1-GHz. To improve the resolution, another scheme is proposed with the RF waveform multiplexed to a dispersive stretched optical pulse, and then sampled in the spectral domain by an optical channelizer with 25-GHz channel spacing. According to the Nyquist’s law and the RF spectrum can be precisely retrieved from the sampling data through an FFT manipulation [13]. The resolution is improved to hundreds of megahertz or even tens of megahertz, while sacrificing the acquisition frame rate due to the post processing based on digital signal processor. Time stretch attracted wide attention recently for its unique of fast continuous single-shot measurement [14]. For the application of the analog-to-digital conversion [15], the electrical signal is intensity modulated on a chirped pulse, and followed with another spool of dispersive fiber, the envelope of the electrical signal is temporally stretched and can be captured with lower bandwidth. However, the time-bandwidth product is degraded as well, since its temporal magnification process do not improve the resolution while greatly enlarge the temporal window. Moreover, an FFT manipulation is required for the spectral analysis, which further hinder the operation speed. This concept combined with compressed sensing technology can further reduce the acquisition bandwidth, e.g. using <1% of the Nyquist sampling rate [20], which is of great importance for ultra-wideband signal. However, this scheme is not suitable for the observation of some fast chirped frequency components, and the complex reconstruction algorithm also hindered its wide application.

Moreover, the all-optical Fourier transform approach multiplexes the RF source to the Fourier domain of an optical pulse by an EOM, followed by a dispersive Fourier transform, its frequency information will be converted to the time domain, proportional to the convolution of the pulse and the scaled RF spectrum. It is noted that 1-GHz frequency only corresponds to 8-pm optical spectral width, in order to obtain finer resolution, a picosecond or even femtosecond pulse source as well as large volume of dispersion are employed here. However, another problem brought by the ultrashort pulse source is that the output field is far beyond the observation bandwidth of the state-of-the-art photo-detector (PD) and oscilloscope, so that the spectral resolution is greatly degraded [16]. Otherwise, some nonlinear gating methods like the autocorrelation detection technique is required, which would greatly hinder the operation speed [17]. To overcome this limitation, a temporal magnification system is introduced in this paper to further stretch the time axis and relax the bandwidth requirement. With suitable dispersion relation, the temporal magnification system is capable of scaling the time axis by hundreds of times [21,22]. Therefore, the aforementioned ultrafast output field can be directly measured in real-time, and this ESA system can easily achieve over MHz acquisition frame rate, leveraging the dispersive frequency-to-time mapping and the time-resolved detection [23]. The ultrafast spectrum analyzer proposed here provide an alternative solution for some applications where ultrafast acquisition frame rate and large observation bandwidth are demanded simultaneous, such as the observation of dynamic frequency evolutions, the capture of instantaneous frequency, etc.
2. Principle of operation

Fig. 1. Schematic of the proposed ultrafast ESA. (a) The temporal ray diagram of the system. A point light source (pulse source) is diverged by propagating a certain distance, before diffracted by a sinusoidal grating (different density corresponds to different RF frequency), and followed by an opposite propagation. Finally, the diffracted spots are amplified by a magnification system. (b) The temporal counterpart of the system. A time-lens is introduced to implement the temporal magnification system, which helps a microwave photonics based spectrum analyzer achieve high frame rate without degrading the resolution.

Figure 1(a) shows the temporal ray diagram of the proposed scheme for RF frequency measurement. A point light source is first transformed to the Fourier domain by propagating a certain distance before periodically modulated by a sinusoidal grating in the Fourier plane. After opposite propagation (opposite direction with identical distance), namely an inverse Fourier transform, the modulated field will be focused into two spots, and the departure of them is proportional to the density of the grating, according to the convolution theorem of Fourier transform. To further separate the two spots, a converging lens is employed here to construct a magnification system. According to the space-time duality [24], the temporal counterpart of this system is proposed as shown in Fig. 1(b). Firstly, an ultrashort optical pulse $A_0(\tau) (U_0(\omega) \text{ in frequency domain})$ is stretched by a spool of dispersion-compensating fiber (DCF), and the output field $A_1(\tau) (U_1(\omega) \text{ in frequency domain})$ can be expressed in frequency domain as $U_1(\omega) = U_0(\omega)G_1(\omega)$, where $G_1(\omega) = \exp(\text{i} \beta_2^{(1)} L_1 \omega^2 / 2)$ is the frequency-domain transfer function of the DCF, with $\beta_2^{(1)}$ and $L_1$ being the group velocity dispersion (GVD) parameter and the fiber length, respectively. Followed by an EOM, the RF source $f(\tau) (F(\omega) \text{ in frequency domain})$ to be measured is uploaded to the optical band. In order to ensure the linear modulation, the bias should be set around $V_\pi$ (switching voltage of the EOM) and the voltage of the RF source should be smaller enough. Assuming that $V_{\text{bias}} = V_\pi$ and $|f(\tau)|$ is sufficiently small (small signal approximation), the modulated field can be approximated as:

$$A_2(\tau) = A_1(\tau) \times \cos \left( \frac{\pi}{2V_x} \left( V_{\text{bias}} + f(\tau) \right) \right) = -\frac{\pi}{2V_x} A_1(\tau) \times f(\tau) \quad (1)$$

After passing through another spool of single-mode fiber (SMF) (frequency-domain transfer function is $G_2(\omega)$), the modulated field is focused to be:

$$A_3(\tau) = \mathcal{F}^{-1} \left\{ -\frac{1}{4V_x^2} \left[ U_0(\omega) G_1(\omega) \otimes F(\omega) \right] G_2(\omega) \right\}$$

$$= \frac{1}{8\pi V_x^2 \Phi_0} \exp \left( \frac{\text{i} \tau^2}{2\Phi_0} \right) \times A_0(\tau) \exp \left( -\frac{\text{i} \tau^2}{2\Phi_0} \right) \otimes F \left( -\frac{\tau}{\Phi_0} \right) \quad (2)$$
where \( \Phi_1 = \beta_2^{(1)}L_1 \) and \( \Phi_2 = \beta_2^{(2)}L_2 \) are the group-delay dispersion (GDD) of the DCF and the SMF respectively, and satisfy: \( \Phi_2 = -\Phi_1 = \Phi_0 \). The output field can be expressed as the convolution of the original ultrashort pulse and the scaled RF spectrum, in other words, a Fourier transform of the RF source is implemented in the optical domain. For an RF source with a single frequency of \( \omega_0 \) and amplitude of \( a \), namely \( f(t) = a \cos(\omega_0 t) \), the output intensity can be calculated as:

\[
I_\tau(\tau) = \frac{a^2}{64 V_x^2} \left[ I_0(\tau + \Phi_0 \omega_0) + I_0(\tau - \Phi_0 \omega_0) \right] \tag{3}
\]

where \( I_0 \) represents the intensity profile of the original pulse (a DC component, namely a pulse located at the zero point of the time axis will appear when \( V_{bias} \) is deviated from \( V'_3 \)). This derivation indicates that the RF frequency is converted to the deviation of the pulse with a mapping relationship of \( \tau = \Phi_0 \omega_0 \). Based on this conversion, the frequency of the RF source can be characterized just by measuring the position of the pulse, with a spectral resolution of \( \delta \Phi = \delta \tau (2\pi \Phi_0) \), where \( \delta \tau \) is the pulsewidth of the output pulse, which is identical to that of the original pulse. It shows that an ultrashort pulse and large dispersion are required to achieve finer resolution. Unfortunately, the ultrashort pulsewidth is far beyond the capability of the state-of-the-art PD and oscilloscope that result in a poor spectral resolution \([16]\), or some complex methods like the autocorrelation detection were employed to measure the output pulses \([17]\), but greatly degraded the acquisition frame rate. Leveraging the space-time duality, a temporal magnification system is introduced here to stretch the temporal axis, so that the ultrashort pulse will be enlarged to be detectable for the conventional oscilloscope. As show in Fig. 1(b), the temporal magnification system is consist of a spool of SMF, a time-lens and a spool of DCF, where \( \Phi_{in} = \beta_2^{(2)}L_{in} \) and \( \Phi_{out} = \beta_2^{(1)}L_{out} \) correspond to the input and the output GDDs, respectively. Similar to the space-lens, the time-lens introduces a quadratic phase modulation in the time axis, and it can be implemented by a variety of methods \([25]\). Considering the four-wave mixing (FWM) based time-lens, an ultrashort pulse first passes through a dispersive fiber to generate the swept-pump. When the bandwidth is wide enough, the amplitude envelop can be neglected, and the swept-pump can be expressed as \( A_p(\tau) = \exp(-i\tau^2/2\Phi_p) \tag{26} \), where \( \Phi_p \) is the GDD of the dispersive fiber. On the other hand, the focused time-lens introduces a quadratic phase modulation in the time axis, and it can be implemented by a variety of methods \([25]\). Considering the four-wave mixing (FWM) based time-lens, the input field of the temporal magnification system, it is first diverged by the input GDD (\( G_{in}(\omega) \) as the transfer function) and becomes \( A_{in}(\omega) \). The parametric mixing process happens between the signal \( A_{in}(\omega) \) and the swept pump \( A_p(\omega) \) in a high nonlinear dispersion-shifted fiber (HNL-DSF), the quadratic phase of the pump will be multiplied to the newly generated idler. The idler can be simply written as \( A_I(\omega) = A_{in}^*(\omega) A_p^*(\omega) = A_{id}(\omega) h(\omega) \tag{27} \), where \( h(\omega) = \exp(-i\tau^2/\Phi) \) is the transfer function of the time-lens, with \( \Phi_f = \Phi_f/2 \) being its focal GDD. Therefore, the scaled output field of the magnification system can be derived as:

\[
A_{out}(\tau) = 3^{-1} \left\{ \left[ U^{-1}_f (-\omega) G_{in}^*(\omega) \otimes H(\omega) \right] \cdot G_{out}(\omega) \right\} \tag{4}
\]

where \( H(\omega) \) is the Fourier transform of \( h(\omega) \), \( G_{out}(\omega) \) is the transfer function of the output GDD. When the temporal imaging condition is satisfied: \( 1/\Phi_{in} - 1/\Phi_{out} = 1/\Phi_f \). Equation (4) can be further simplified as:

\[
A_{out}(\tau) = \frac{1}{\sqrt{M}} \exp \left[ -\frac{i\tau^2}{2M \Phi_f} \right] \left[ A_i^*(\tau) \cdot \frac{\tau}{\Phi_f} \right] \tag{5}
\]
where $M = \Phi_{\text{out}} / \Phi_{\text{in}}$ is the magnification factor. The corresponding output intensity is:

$$I_{\text{out}}(\tau) = I_0 \left( \frac{\tau}{M} + \Phi_0 \omega_0 \right) + I_0 \left( \frac{\tau}{M} - \Phi_0 \omega_0 \right)$$

(6)

It is indicated that the output field is scaled by a factor of $M$. By adjusting the dispersive parameters of the temporal magnification system, the time axis can be stretched by tens of times, and the conventional PD and oscilloscope are able to capture the output field. Thus an ultrafast RF spectrum analysis approach is realized without degrading the resolution. According to the derived spectral resolution expression, 1-ps pulsewidth ($\delta \tau$) and 1ns/nm dispersion ($\Phi_0$) indicated a resolution of about 200 MHz. The simulation results of this ultrafast ESA is shown as Fig. 2(a), with the ideal temporal magnification system, it can resolve frequencies with 200-MHz separation (red line). If consider the limited temporal window of the magnification time-lens, the resolution will be a little bit degraded (blue line). While the high-order dispersion will further degraded as the black line, which can only resolve 500-MHz frequency spacing. Another important feature of this ultrafast ESA is its ability to resolve some fast chirped frequencies, and its simulation performance is manifested in Fig. 2(b). As the frequency sweeping increasing, the resolving spectral width is broadened, and this scheme can capture the maximum chirp rate of 40 MHz/ns, with the peak power degraded by a factor of 2.

Fig. 2. Simulation results of the system. (a) Resolution performance under ideal conditions (red), limited time-lens window (blue), high order dispersions (black). (b) Measurement results of chirped frequency with chirp rate changing from 0 to 80 MHz/ns, with 20 MHz/ns spacing.

3. Experimental results and discussions

The experimental setup of the ultrafast and large bandwidth ESA is illustrated in Fig. 3, and it consists of the optical Fourier transform part and the temporal magnification part. To make sure the repetition rates are synchronized, the pulse sources of these two parts are filtered from the same mode-locked fiber laser (MLFL), with 1-ps pulsewidth and 50-MHz repetition rate. The spectrum and the normalized waveform (seriously broadened due to the limited observation bandwidth) of the MLFL are shown in Fig. 4. To generate a swept pump of the
time-lens, part of the pulse source (the lower branch, filtered from 1555 nm to 1565 nm) passes through a spool of 5-km SMF. While the upper branch of the pulse source (filtered from 1532 nm to 1544 nm) is applied for the optical Fourier transform part, which is first stretched by a spool of DCF with ~1-ns/nm dispersion (compensating 60-km SMF). Then the RF signal under test was multiplexed onto the wavelength-to-time mapped stretched source through an amplitude modulator, with 20-GHz bandwidth and 3.5-V switching voltage. The small signal approximation requires the drive voltage under 1 V. This modulator is followed with 63-km SMF, 60-km of which is matched with the dispersion of the first DCF, and compresses the stretched source to realize the optical Fourier transform part. Another 3-km SMF is acted as the input dispersion of the temporal magnification system. Subsequently, the signal is coupled into a 100-m HNL-DSF with the lower branch swept-source as the time-lens. The generated idler is filtered out, and passing through a spool of DCF as the output dispersion. Finally, it is boosted up by a pre-amplifier and captured by a 40-GHz PD and a 16-GHz real-time oscilloscope. It is emphasized that the fiber length of the time-lens system is optimized to satisfy the temporal imaging condition, while ensuring a large magnification factor to stretch the pulse to be directly captured by the conventional temporal oscilloscope.

A 10-GHz sinusoidal signal under test is first applied to the modulator with 2.5-V bias voltage. The waveform before (average power of 8.5 dBm) and after (average power of –3.5 dBm) the amplitude modulator is exhibited in Fig. 5, with ~12-dBm insertion loss. As shown in Fig. 5(a), the ultrashort pulse source is temporally stretched to around 13 ns, which represents the time window of this ultrafast ESA system. Considering the limited time window, this ESA is incapable of measuring a frequency below 70 MHz, though it can be further improved by increasing the dispersion (> 1 ns/nm) of the optical Fourier transform part or providing larger spectral width. Figure 5(b) certified that the RF frequency is successfully multiplexed to the time axis of the stretched field.
Fig. 5. The normalized temporal waveforms before (a) and after (b) the amplitude modulator. The temporal shape with 13-ns duration resembles its spectrum. The inset shows the zoom-in fringes after the modulator with 10-GHz sinusoidal signal.

After the 63-km SMF, the signal is coupled together with the lower branch swept pump into the HNL-DSF, where the FWM process took place. With 15-dBm pump power and 5-dBm signal power, the FWM process achieves −20-dB conversion efficiency, as shown in Fig. 6(a). The idler was filtered out and passed through the output dispersion (DCF), and the output trace was captured as shown in Fig. 6(b), where 50-MHz acquisition frame rate is realized. Within a single observation period, as shown in Fig. 6(c), the central pulse represents the DC component, which can be adjusted by controlling the bias voltage, and it is set as the reference frequency. Besides the DC pulse, the other two neighboring pulses represent the frequency of the RF signal, also characterized by a conventional ESA (inset of Fig. 6(c)). Some harmonic frequency components appear due to the large drive voltage (2.2 V in this configuration) breaking the small signal approximation, as well as the FWM process. It is also noted that, the harmonic can be suppressed as the drive voltage decrease, and by adjusting the $V_{bias}$, the DC component can be controlled accordingly, as shown in Fig. 6(d).

Fig. 6. (a) The spectra before (blue dash-dotted line) and after (red solid line) FWM process. (b) The real-time acquisition of the RF spectrum with a 10-GHz sinusoidal signal under test, it achieves 50-MHz acquisition frame rate. (c) Single period of (b), with inset exhibited the tested spectrum. (d) Under 1 V drive voltage, the experiment results with different bias voltages (blue: 3.2 V; red: 3.5 V).
Scanning the frequency of the RF signal from 2 GHz to 20 GHz with an equal spacing of 2 GHz, the results recorded by the oscilloscope is exhibited in Fig. 7(a), where different colors represent different response frequencies. The amplitude roll-off curve mainly comes from the limited EOM bandwidth, the pump pulse shape, and FWM conversion bandwidth. Also the temperature fluctuation will introduce some misalignment between the pump and signal pulse in the FWM processing, thus lead to a time-dependent amplitude fluctuation. Owing to the third-order dispersion in the fibers, there is a notable asymmetry between the two neighboring pulses under the same RF frequency. Since the right hand side part is much sharper than the left part, only the right hand side is considered to achieve finer resolution. Considering the frequency-to-time mapping ratio is 4 GHz/ns, 250-ps pulsewidth corresponds to 1-GHz RF spectral resolution. According to the time-bandwidth product limit, the idea resolution of the proposed ultrafast ESA will be 50 MHz, by making full use of the 20-ns temporal window (with ideal dispersion). However, in practical, the resolution will be seriously degraded by the higher-order dispersion and the dispersion mismatch in the temporal magnification system, as well as the partially occupied temporal window [28,29].

Finally, the dynamic range performance of the ultrafast ESA system is investigated by successively increasing the amplitude of the RF signal before the drive amplifier, and the normalized response power (the maximum peak power as the reference) of the output pulse is depicted in Fig. 7(b). It is evident that the modulator has an optimum operating range, namely the small signal approximation, where the output power of the ultrafast ESA is quasi-linear increasing, as the Eq. (3). In fact, a low driving voltage would result in poor signal-to-noise ratio, while the higher driving voltage exceeds the small signal approximation range of the EOM, and both cases will confine the dynamic range. Therefore, suitable pre-amplification is necessary before measuring an RF signal using the ultrafast ESA proposed here.

4. Conclusion

In conclusion, we have experimentally demonstrated an ultrafast and large bandwidth ESA. Under the condition of the experimental parameters, the spectral resolution can theoretically achieve hundreds of megahertz, furthermore, it will be enhanced if a shorter pulse source or larger dispersion is employed. But actually, the experimental result shows that the minimum resolvable frequency spacing of the analyzer is about 1-GHz, which can be greatly improved when a more accurate dispersion matching is implemented. Additionally, a larger pupil size of the time-lens, or equally a longer duration of the swept pump here, brings a finer resolution. Moreover, the demonstration confirmed an observation bandwidth over 20 GHz. Due to the limitations of the equipments, a higher frequency is not tested in the experiment.
In fact, the measurement range of the analyzer can be increased by using a time-lens with a larger aperture, but finally limited by the bandwidth of the modulator. Specially, an acquisition frame rate of 50 MHz is demonstrated here, which can be further raised up if necessary. Such an ultrafast operation speed of the RF spectrum analyzer is of great significance for various applications such as dynamic spectrum monitoring and instantaneous frequency capturing.

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